



Concrete, Pictorial and Abstract Calculation Policy

Recommended practice delivering a mastery approach

True mastery aims to develop all children's mathematical understanding at the same pace. As much as possible, children should be accessing the same learning. Differentiation should primarily be through support, scaffolding and deepening, not through task.

Consistency in language is essential for pupils to understand the concepts presented in mathematics. If other, 'child-friendly' terminology is used, this must be alongside the current terminology recommended by maths specialists. Using this will support children with their examinations and throughout secondary school.

Evidence repeatedly shows that mixed ability seating increases less confident pupils' perception of mathematical capability, which impacts positively upon outcomes.

Concrete, pictorial, abstract (CPA) concepts should not be confused as differentiation for lower, middle, higher attaining children. CPA is an approach to be used with the whole class and teachers should promote each area as equally valid.

Times tables

Times tables to be taught formally for 5 to 10 mins at the beginning of each maths lesson. A demonstration of the approach can be found below:

<https://www.youtube.com/watch?v=yXdHGBfogfw>

Years 1 – 2, 5 and 10

Year 2 – 2, 5 and 10 including division facts

Year 3 – 2, 5, 10, 3, 4 and 8 including division facts

Years 4 to 6 – all times tables including division facts (12 x 12)

Within key stage 2, children to develop fluency and variation of times table based upon known facts as appropriate to year group expectation for multiplication (see grid multiplication e.g year 3 – multiples of 10 x by digit to support 2 digit x single digit multiplication):

e.g

$$4 \times 3 = 12$$

So

$$40 \times 3 = 120$$

$$4 \times 30 = 120$$

$$400 \times 3 = 1200 \text{ etc}$$

And

$$0.4 \times 3 = 1.2$$

$$4 \times 0.3 = 1.2$$

$$0.04 \times 3 = 0.12$$

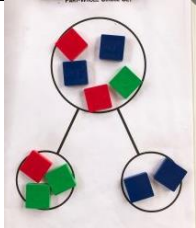
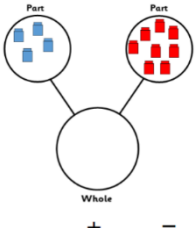
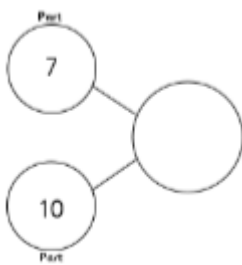


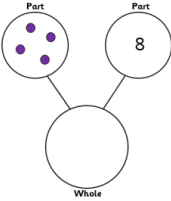

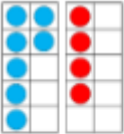
etc

Children to be tested weekly on times table for that week.

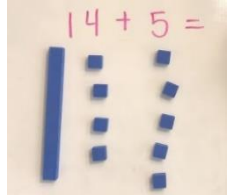

Representations used by Year Group (* INDICATES USE)

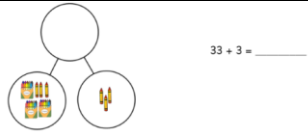

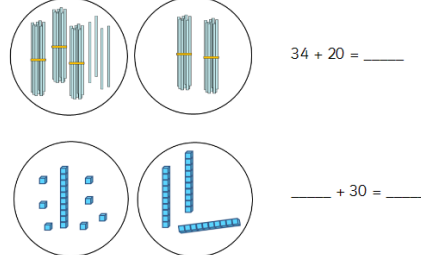
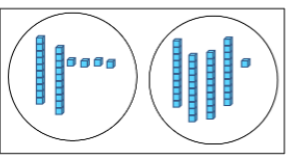
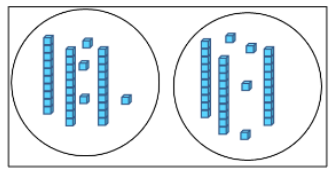
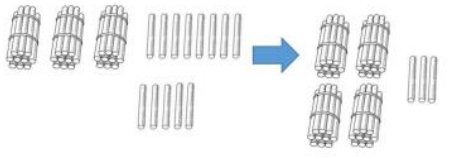
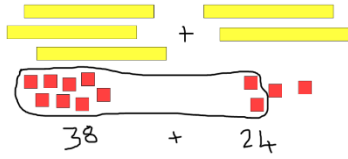
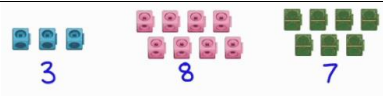
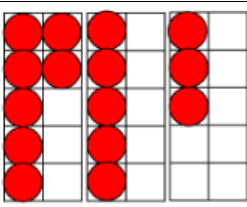
	Tens Frame	Part Whole	Sticks and Dots	Number Lines	Place Value	Bar Models
EYFS	*	*				
Y1	*	*	*	*		
Y2	*	*	*	*		*
Y3	*	*	*	*	*	*
Y4	*	*	*	*	*	*
Y5	*	*	*	*	*	*
Y6	*	*	*	*	*	*

Year 1 Addition

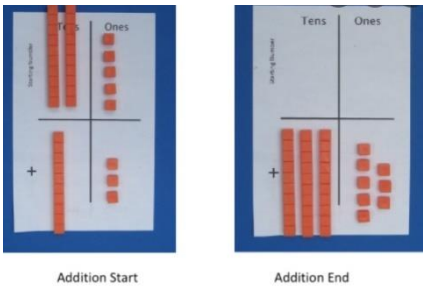
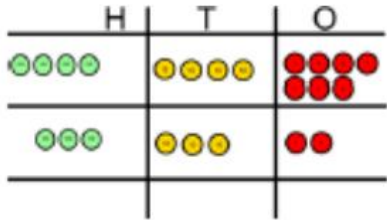
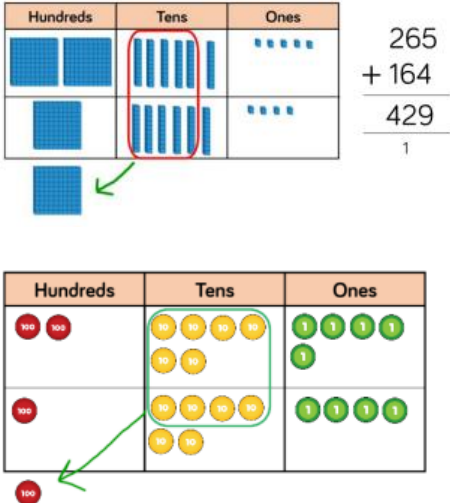
Objective	Concrete	Pictorial	Abstract
Combining two parts to make a whole – part, part, whole.	 <p>Use of cubes or other manipulatives.</p>	 <p>Simple pictures or symbols to represent numbers.</p> <p>___ + ___ = ___</p>	
Starting with the bigger number and counting on.	 <p>Starting with the bigger number and counting on 1 by 1.</p>	 <p>Larger part identified.</p> 	<p>$7 + 5 = \underline{\quad}$</p> <p>$\underline{\quad} = 7 + 6$</p>
Regrouping to make 10.	 <p>Start with bigger number and use number bonds to make ten.</p>	 <p>$7 + 4 = \underline{\quad}$</p> <p>$7 + 3 + 1$</p> <p>$10 + 1 = \underline{\quad}$</p>	<p>$4 + 7 = \underline{\quad}$</p> <p>$\underline{\quad} + \underline{\quad} + \underline{\quad}$</p> <p>$\underline{\quad} + \underline{\quad} = \underline{\quad}$</p>

Year 2 Addition

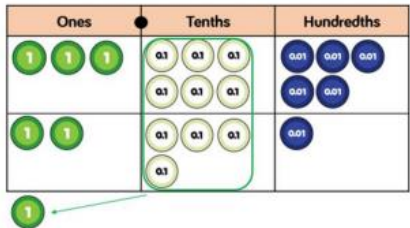
Addition of 1 digit number to a 2 digit number	 <p>Use of base ten, tens frames and straws.</p>	 <p>How many altogether?</p>	$54 + 4 =$
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		 $33 + 3 = \underline{\quad}$	
Adding multiples of ten to a 2 digit number.	 $25 + 10 = 35$	 $34 + 20 = \underline{\quad}$ $\underline{\quad} + 30 = \underline{\quad}$	$23 + 20 = \underline{\quad}$ $52 + 30 = \underline{\quad}$ $\underline{\quad} = 50 + 48$
Addition of two 2 digit numbers (without bridging 10)	 $24 + 41 = \underline{\quad}$	<p>Use of base 10 and stick bundles</p>  $\underline{\quad} + \underline{\quad} = \underline{\quad}$	$23 + 13 =$ $23 + 10 + 3 =$ $33 + 3 = 36$
Addition of 1 digit number to a 2 digit number (bridging 10)	 <p>Use of manipulatives (straws, tens frames and base 10)</p> <p>Regroup of ten.</p>	 $38 + 24 = 62$ <p>Children to draw the regrouped ones to form a ten then add the remaining ones.</p>	$46 + 7$ $36 + 65$
Addition of three single digit numbers	 $3 + 8 + 7 = 18$ <p>Children to form groups of ten first and then add third number.</p>	 $7 + 5 + 3 = \underline{\quad}$	$9 + 5 + 1 = \underline{\quad}$ $3 + 5 + 7 = \underline{\quad}$

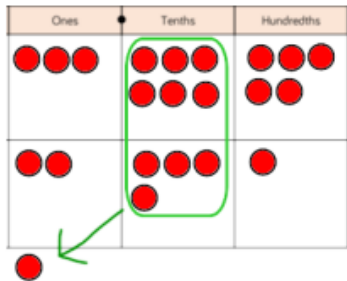
Year 3 Addition

<p>Column addition without exchange</p>	<div data-bbox="645 108 1066 395">  <p>25 + 13</p> <p>Moving onto use of place value counters</p> </div>	<div data-bbox="1167 124 1552 344">  </div> <p>Display the column written method alongside the place value chart representation.</p>	<div data-bbox="1653 116 1946 373"> $\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$ </div> <p>Adding the ones then the tens and then the hundreds.</p>
<p>Column addition with exchange</p>	<div data-bbox="629 549 1077 1054">  </div>	<p>As concrete model using printed place value charts and children representing numbers on charts with circles.</p>	<div data-bbox="1666 572 1904 916"> $\begin{array}{r} 265 \\ + 164 \\ \hline 429 \\ 1 \end{array}$ </div>
<p>Year 4 to 6 Addition</p>			
<p>Addition of two 4-digit numbers or greater.</p>	<p>Children will continue to add increasingly larger numbers as per year 3 guidance Year 4 – up to four digits Year 5 and 6 - more than four digits.</p>		

Adding decimals



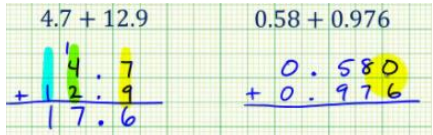
Use of place value charts



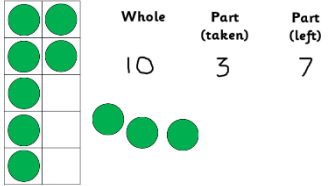
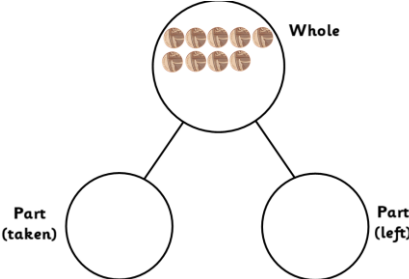
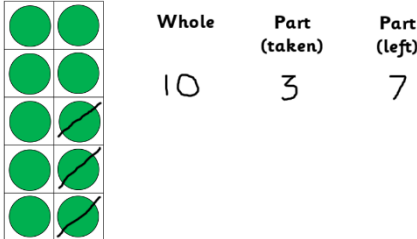
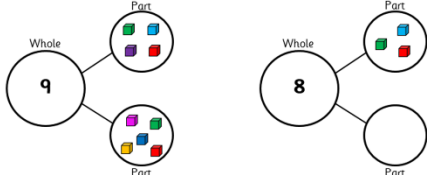

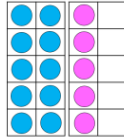
Drawing own representations

$$\begin{array}{r} 3.65 \\ + 2.41 \\ \hline 6.06 \\ \hline 1 \end{array}$$

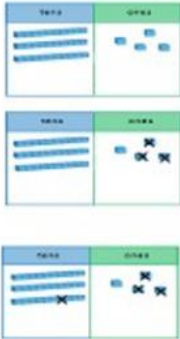

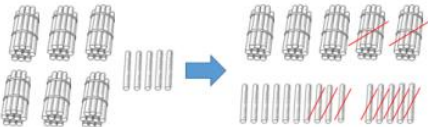
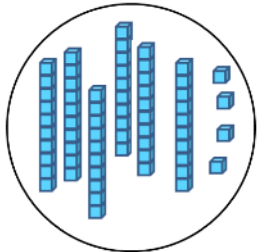

When adding numbers with different amounts of decimal places, ensure that place holders are included.



Year 1 Subtraction

Objective	Concrete	Pictorial	Abstract
<p>Taking away ones</p> <p>(Link with addition and number bonds used in part whole models to show inverse)</p>	<p>What does this show?</p>  <p>Whole 10 Part (taken) 3 Part (left) 7</p> <p>Physical objects removed from a whole. Use of tens frame and part whole model.</p> 	<p>What does this show?</p>  <p>Whole 10 Part (taken) 3 Part (left) 7</p> <p>Crossing out of the part to be taken away.</p>  <p>____ - ____ = ____ ____ - ____ = ____</p> <p>Link to part whole model as inverse of addition.</p>	$10 - 4 = \underline{\quad}$ $8 - \underline{\quad} = 3$
<p>Making 10</p>	 <p>$14 - 6 =$</p> <p>Take away the four to make ten and then the other two counters.</p>	 <p>$15 - 7 = \underline{\quad}$</p> <p>$15 - 5 - 2 = \underline{\quad}$</p> <p>$10 - \underline{\quad} = \underline{\quad}$</p>	<p>$14 - 6 = \underline{\quad}$ $13 - 9 = \underline{\quad}$</p> <p>$14 - \underline{\quad} = \underline{\quad}$ $\underline{\quad} - \underline{\quad} = \underline{\quad}$</p> <p>$10 - \underline{\quad} = \underline{\quad}$ $\underline{\quad} - \underline{\quad} = \underline{\quad}$</p>

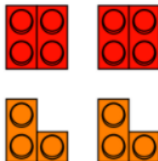
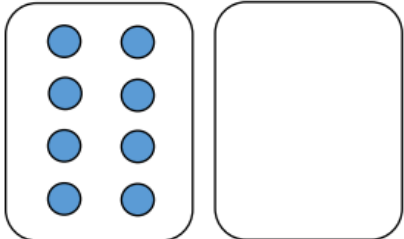
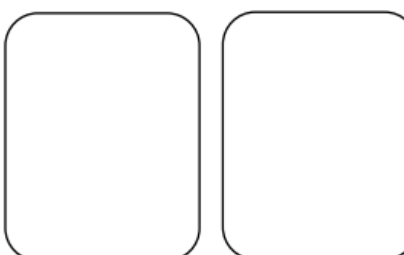


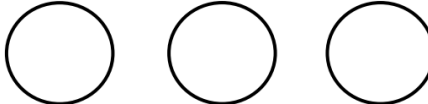
Year 2 Subtraction

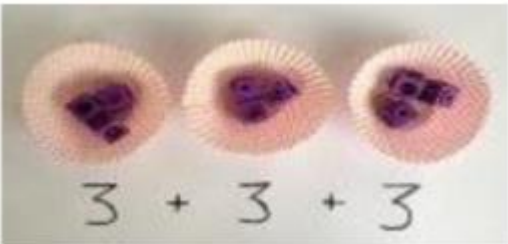


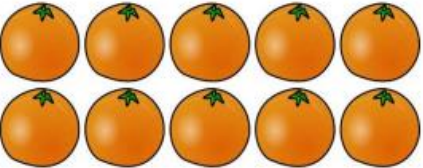
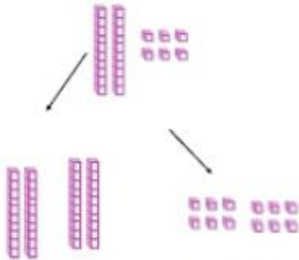
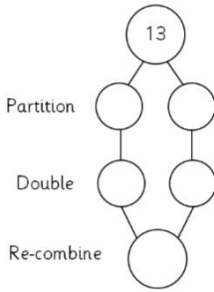
<p>Subtracting a 1-digit or 2-digit number from a 2-digit number without exchanging</p>	<p>$34 - 13 = 21$</p>  <p>Use Dienes to show how to partition the number when subtracting without regrouping.</p>	 <p>$43 - 21 = 22$</p> <p>Children to draw their own representation of base ten.</p>	<p>$43 - 21 = 22$</p>
<p>Subtracting using exchange</p>	 <p>Children to be made aware that bundles may need to be separated to be able to subtract.</p> <p>This moves on to the use of base 10 equipment.</p>  <p>$64 - 15 = \underline{\quad\quad}$</p>	<p>$23 - 16 = 7$</p>  <p>Children to draw their own representation of base ten and then cross out accordingly.</p>	<p>$23 - 16 = 7$</p>

Year 3 Subtraction

Subtraction without exchange	<div><table><tr><th>H</th><th>T</th><th>O</th></tr><tr><td></td><td></td><td></td></tr></table><div>$\begin{array}{r} 133 \\ - 21 \\ \hline \end{array}$<p>Children to use base 10 and place value charts and then move on to the place value counters.</p></div></div>	H	T	O				<div></div> <div><p>Calculations</p>$176 - 64 =$$\begin{array}{r} 176 \\ - 64 \\ \hline 112 \end{array}$</div>	<div>$\begin{array}{r} 176 \\ - 64 \\ \hline 112 \end{array}$</div>
H	T	O							
Subtraction involving exchange	<div><table><tr><th>H</th><th>T</th><th>O</th></tr><tr><td></td><td></td><td></td></tr></table><div>$\begin{array}{r} 133 \\ - 26 \\ \hline \end{array}$<p>Children to use base 10 and place value charts and establish concept on exchange then move on to the place value counters.</p></div></div>	H	T	O				<div>$\begin{array}{r} 3 \quad 1 \\ 435 \\ - 273 \\ \hline 262 \end{array}$</div> <p>Draw own representations to support calculation.</p>	<div>$\begin{array}{r} 3 \quad 1 \\ 435 \\ - 273 \\ \hline 262 \end{array}$</div>
H	T	O							
Year 4-6 Subtraction									
Subtraction of two 4-digit numbers or greater.	Children will continue to subtract increasingly larger numbers as per year 3 guidance Year 4 – up to four digits Year 5 and 6 - more than four digits.								
Subtracting decimals	<div><table><tr><th>Ones</th><th>Tenths</th><th>Hundredths</th></tr><tr><td></td><td></td><td></td></tr></table><p>Use of place value charts</p></div>	Ones	Tenths	Hundredths				<div></div> <p>Draw own representation</p>	<div>$\begin{array}{r} 4 \quad 1 \\ 5.43 \\ - 2.7 \\ \hline 2.73 \end{array}$</div>
Ones	Tenths	Hundredths							

Year 1 Multiplication

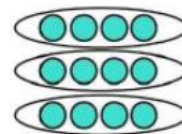
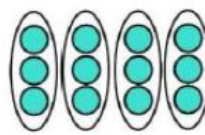
Objective	Concrete	Pictorial	Abstract
Doubling	 $4 + 4 = \square$ $2 \times 4 = \square$ $3 + 3 = \square$ $2 \times 3 = \square$ Use of manipulatives to demonstrate doubling	<p>Double 8 is _____</p>  <p>Double 6 is _____</p> 	$8 \times 2 =$ $6 \times 2 =$
Making equal groups and counting the total.	 <p>Use manipulatives to make equal groups and then count up</p>	<p>There are _____ flowers in each of _____ pots.</p>  <p>How many flowers are there altogether? _____</p>	<p>There are two bananas in each bowl. How many bananas are there altogether? _____</p>  <p>$3 \times 2 =$</p>

Repeated addition	 <p>Different objects used to add equal groups</p>	 <p>There are 5 chocolates in each row. There are 4 rows. How many chocolates altogether?</p> <p>_____ chocolates</p>	$5 \times 4 =$ $5 + 5 + 5 + 5 =$
Use of arrays	 <p>Use manipulatives to create arrays to represent multiplication then count.</p>	 <p>$5 \times 2 =$</p>	$5 \times 2 =$
<h2 style="text-align: center; color: yellow; background-color: #00a0e3; padding: 5px;">Year 2 Multiplication</h2>			
Doubling	<p>Physical doubling with objects to represent a single digit moving to two digit numbers using base 10.</p>  <p>$40 + 12 = 52$</p>	<p>Draw representations of their own to show the doubling and use of sticks and dots to represent base 10.</p>	 <p>3. Double 13 is _____.</p> <p>Moving onto: Double 23 and $23 \times 2 =$</p>

Use of arrays to show that multiplication is commutative



Creating arrays using counters or other objects.



Explore arrays to create calculation and show the commutativity.

Use an array to write multiplication sentences and reinforce repeated addition.



$$5 + 5 + 5 = 15$$

$$3 + 3 + 3 + 3 + 3 = 15$$

$$5 \times 3 = 15$$

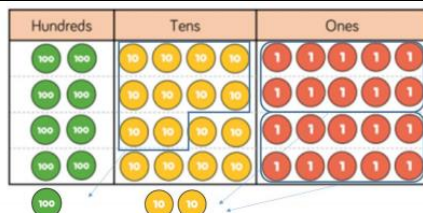
$$3 \times 5 = 15$$

Year 3 and 4 Multiplication

Multiplication of 2 and 3 digit number by a single digit

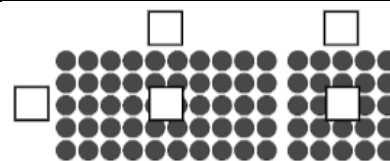
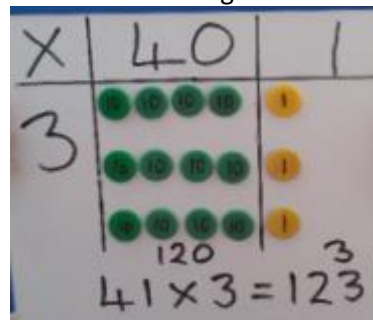
(Year 3 – 2 digit by 1 digit)

(Year 4 – 3 digit by 1 digit)



Use of place value chart using base ten or counters froming the array and then adding from the ones

This builds to the grid method



$$15 \times 5 = \square$$

$$\square + \square = \square$$

Moving to



$$14 \times 3 = \square$$

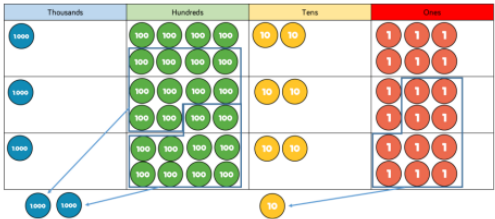
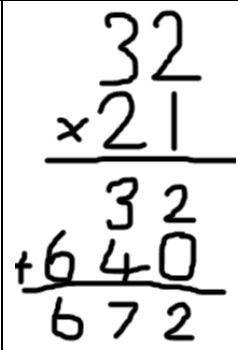
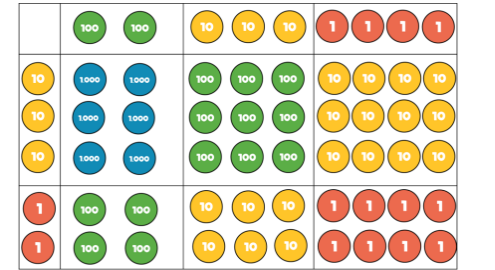
$$\square + \square = \square$$

$$245 \times 4 = 980$$

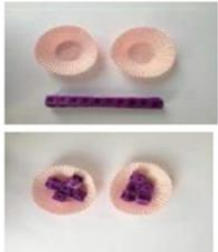
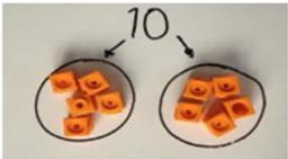
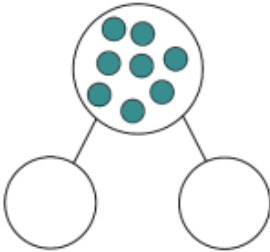


Using formal column method

	H	T	O
	2	4	5
x			4
	9	8	0
	1	2	


Year 5 and 6 Multiplication

<p>Multiplication of 4 digit number by single digit</p> <p>This method to also be used when multiplying decimals</p>	 <p>1826 x 4 =</p> <p>Children to create own array using place value counters on place value chart.</p>	<p>Own representation on place value chart with formal column method alongside.</p>	<div>1,826 × 3 = 5,478</div> <table><tr><th></th><th>Th</th><th>H</th><th>T</th><th>O</th></tr><tr><td></td><td>1</td><td>8</td><td>2</td><td>6</td></tr><tr><td>x</td><td></td><td></td><td></td><td>3</td></tr><tr><td></td><td>5</td><td>4</td><td>7</td><td>8</td></tr><tr><td></td><td>2</td><td></td><td>1</td><td></td></tr></table>		Th	H	T	O		1	8	2	6	x				3		5	4	7	8		2		1																		
	Th	H	T	O																																									
	1	8	2	6																																									
x				3																																									
	5	4	7	8																																									
	2		1																																										
<p>Multiplying a 2 digit number by a 2 digit number</p>	<table><tr><th></th><th>30</th><th>2</th><th></th></tr><tr><td>20</td><td></td><td></td><td>640</td></tr><tr><td>1</td><td></td><td></td><td>32</td></tr></table> <p>32 x 21= 640 + 32 = 672</p> <p>Use of counters on a grid</p>		30	2		20			640	1			32	<table><tr><th>x</th><th>30</th><th>2</th></tr><tr><td>20</td><td></td><td></td></tr><tr><td>1</td><td></td><td></td></tr></table>	x	30	2	20			1																								
	30	2																																											
20			640																																										
1			32																																										
x	30	2																																											
20																																													
1																																													
<p>Multiplying a 3 digit number by a 2 digit number</p>	 <p>Use of counters on grid (Values can be written as numerals rather than place value)</p>	<table><tr><th>x</th><th>200</th><th>30</th><th>4</th></tr><tr><td>30</td><td>6,000</td><td>900</td><td>120</td></tr><tr><td>2</td><td>400</td><td>60</td><td>8</td></tr></table>	x	200	30	4	30	6,000	900	120	2	400	60	8	<table><tr><th></th><th>Th</th><th>H</th><th>T</th><th>O</th></tr><tr><td></td><td></td><td>2</td><td>3</td><td>4</td></tr><tr><td>x</td><td></td><td></td><td>3</td><td>2</td></tr><tr><td></td><td></td><td>4</td><td>6</td><td>8</td></tr><tr><td>17</td><td>1</td><td>0</td><td>2</td><td>0</td></tr><tr><td>7</td><td>7</td><td>4</td><td>8</td><td>8</td></tr></table>		Th	H	T	O			2	3	4	x			3	2			4	6	8	17	1	0	2	0	7	7	4	8	8
x	200	30	4																																										
30	6,000	900	120																																										
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	Th	H	T	O																																									
		2	3	4																																									
x			3	2																																									
		4	6	8																																									
17	1	0	2	0																																									
7	7	4	8	8																																									


Year 1 Division

Objective	Concrete	Pictorial	Abstract
Division as sharing into equal groups (halving)	 	 $8 \div 2 =$	$8 \div 2 =$
Division as grouping		 <p>Here are 22 seeds. 2 seeds go in each pot How many pots can be filled? _____ pots</p>	$22 \div 2 =$

Year 2 Division

Division as sharing with a remainder	Children to physically share an amount between pots	 <p>10 eggs shared between 4 boxes. How many in each box? ____ How many eggs left over? ____</p> <p>when sharing. Children to move onto drawing own representations</p>	Children to draw and cross out	$13 \div 4 =$
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Division as grouping no remainder



Tom has 10 cakes. He puts 2 cakes in each box.
How many boxes can he fill? _____

Children to physically draw around the groups.

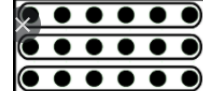
As concrete method.
Children to draw own representations.



$\frac{\quad}{\quad} \div \frac{\quad}{\quad} = \frac{\quad}{\quad}$

$$18 \div 3 = \underline{\quad}$$

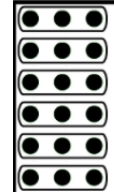
Division as an array




Children to have the arrays to then group

Children to understand the inverse link to multiplication.

$18 \div 3 = 6$



$18 \div 6 = 3$



$\frac{\quad}{\quad} \times \frac{\quad}{\quad} = \frac{\quad}{\quad}$

$\frac{\quad}{\quad} \times \frac{\quad}{\quad} = \frac{\quad}{\quad}$

$\frac{\quad}{\quad} \div \frac{\quad}{\quad} = \frac{\quad}{\quad}$

$\frac{\quad}{\quad} \div \frac{\quad}{\quad} = \frac{\quad}{\quad}$

$$18 \div 3 = \underline{\quad}$$

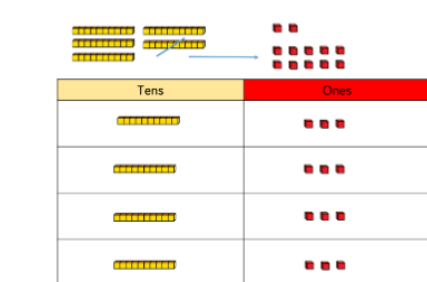
Year 3- 6 Division

The principles apply for division for Years 3 to 6 with ever increasing numbers:

Year 4 – 3 digits by 1 digit

Year 5 and 6 – 4 digits by 1 digit

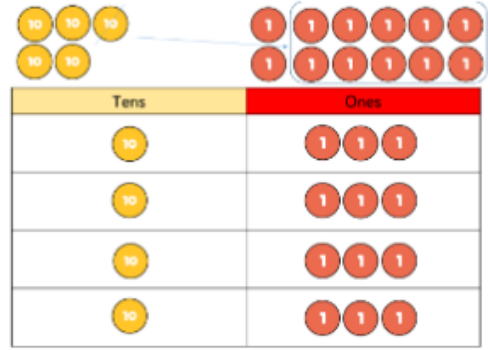
Division by sharing with exchange



Use of

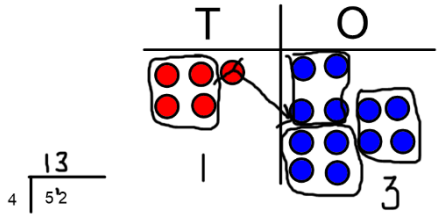
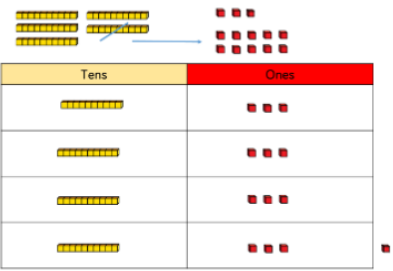
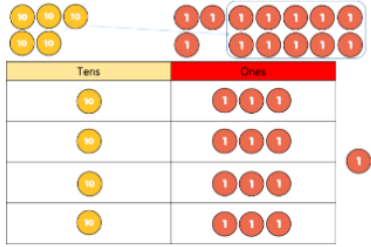
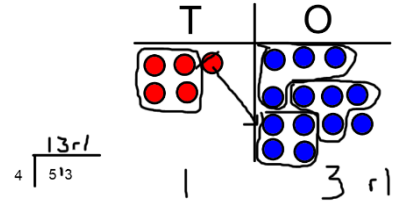
place value charts and base ten and counters physically moving counters and exchanging

Develop own representations using place value charts



$$52 \div 4 = 13$$

Moving to bus stop method

			$\begin{array}{r} 13 \\ 4 \overline{) 52} \end{array}$
Division by grouping with exchange	 <p>Use of place value charts</p>	As previous with children drawing own representation.	$\begin{array}{r} 13 \\ 4 \overline{) 52} \end{array}$
Division by sharing with exchange and remainders	 <p>Use of place value charts and base ten and counters physically moving counters and exchanging</p>	 <p>Develop own representations using place value charts</p>	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> $53 \div 4 = 13 \text{ r}1$ </div> <p>Moving to bus stop method</p> $\begin{array}{r} 13 \text{ r}1 \\ 4 \overline{) 53} \end{array}$
Division by grouping with exchange and remainders	 <p>Use of place value chart</p>	As previous with children drawing own representation.	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> $53 \div 4 = 13 \text{ r}1$ </div> <p>Moving to bus stop method</p>

$$\begin{array}{r} 13r1 \\ 4 \overline{) 513} \end{array}$$

Year 6 Division

Division of multiple digits by a 2-digit number. (Long division)	Concrete resources will not help this skill.	Pictorial representation do nothelp this skill but the writing of key multiple facts will provide a scaffold for children to use.	<div><table><tr><td></td><td></td><td>0</td><td>3</td><td>6</td></tr><tr><td>1</td><td>2</td><td>4</td><td>3</td><td>2</td></tr><tr><td></td><td>-</td><td>3</td><td>6</td><td>0</td></tr><tr><td></td><td></td><td></td><td>7</td><td>2</td></tr><tr><td></td><td>-</td><td></td><td>7</td><td>2</td></tr><tr><td></td><td></td><td></td><td></td><td>0</td></tr></table><div>(x30)</div><div>(x6)</div><div><div>12 × 1 = 12 12 × 2 = 24 12 × 3 = 36 12 × 4 = 48 12 × 5 = 60 12 × 6 = 72 12 × 7 = 84 12 × 8 = 96 12 × 9 = 108 12 × 10 = 120</div><div>432 ÷ 12 = 36</div></div></div>			0	3	6	1	2	4	3	2		-	3	6	0				7	2		-		7	2					0
		0	3	6																													
1	2	4	3	2																													
	-	3	6	0																													
			7	2																													
	-		7	2																													
				0																													
Division of multiple digits by a 2-digit number. (Short division)	Concrete resources will not help this skill.	Pictorial representation do nothelp this skill but the writing of key multiple facts will provide a scaffold for children to use.	<div><table><tr><td></td><td></td><td>0</td><td>3</td><td>6</td></tr><tr><td></td><td>12</td><td>4</td><td>4</td><td>3</td></tr><tr><td></td><td></td><td></td><td>7</td><td>2</td></tr></table><div>12 × 1 = 12 12 × 2 = 24 12 × 3 = 36 12 × 4 = 48 12 × 5 = 60 12 × 6 = 72 12 × 7 = 84 12 × 8 = 96 12 × 9 = 108 12 × 10 = 120</div><div>Children to move to short division method as soon as possible to reduce the need for subtraction where mistakes could arise.</div></div>			0	3	6		12	4	4	3				7	2															
		0	3	6																													
	12	4	4	3																													
			7	2																													